SAMPLE QUESTION PAPER

Class X Session 2024-25

MATHEMATICS STANDARD (Code No.041)

TIME: 3 hours MAX.MARKS: 80

General Instructions:

Read the following instructions carefully and follow them:

1. This question paper contains 38 questions.
2. This Question Paper is divided into 5 Sections A, B, C, D and E.
3. In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are Assertion- Reason based questions of 1 mark each.
4. In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 marks each.
5. In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.
6. In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.
7. In Section E, Questions no. 36-38 are case study based questions carrying 4 marks each with sub parts of the values of 1, 1 and 2 marks each respectively.
8. All Questions are compulsory. However, an internal choice in 2 Question of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
9. Draw neat and clean figures wherever required.
10. Take $\pi = 22/7$ wherever required if not stated.
11. Use of calculators is not allowed.

	Section A	
	Section A consists of 20 questions of 1 mark each.	
1.	The graph of a quadratic polynomial p(x) passes through the points (-6,0), (0, -30), (4,-20) and (6,0). The zeroes of the polynomial are A) - 6,0 B) 4, 6 C) - 30,-20 D) - 6,6	1

Table 2.1

x	-2	-1	0	1	2	3	4	5
$y = x^2 - 3x - 4$	6	0	-4	-6	-6	-4	0	6

If we locate the points listed above on a graph paper and draw the graph, it will actually look like the one given in Fig. 2.2.

In fact, for any quadratic polynomial $ax^2 + bx + c$, $a \neq 0$, the graph of the corresponding equation $y = ax^2 + bx + c$ has one of the two shapes either open upwards like $\cup$ or open downwards like $\cap$ depending on whether $a > 0$ or $a < 0$. (These curves are called **parabolas**.)

You can see from Table 2.1 that -1 and 4 are zeroes of the quadratic polynomial. Also note from Fig. 2.2 that -1 and 4 are the x-coordinates of the points where the graph of $y = x^2 - 3x - 4$ intersects the x-axis. Thus, the zeroes of the quadratic polynomial $x^2 - 3x - 4$ are x-coordinates of the points where the graph of $y = x^2 - 3x - 4$ intersects the x-axis.

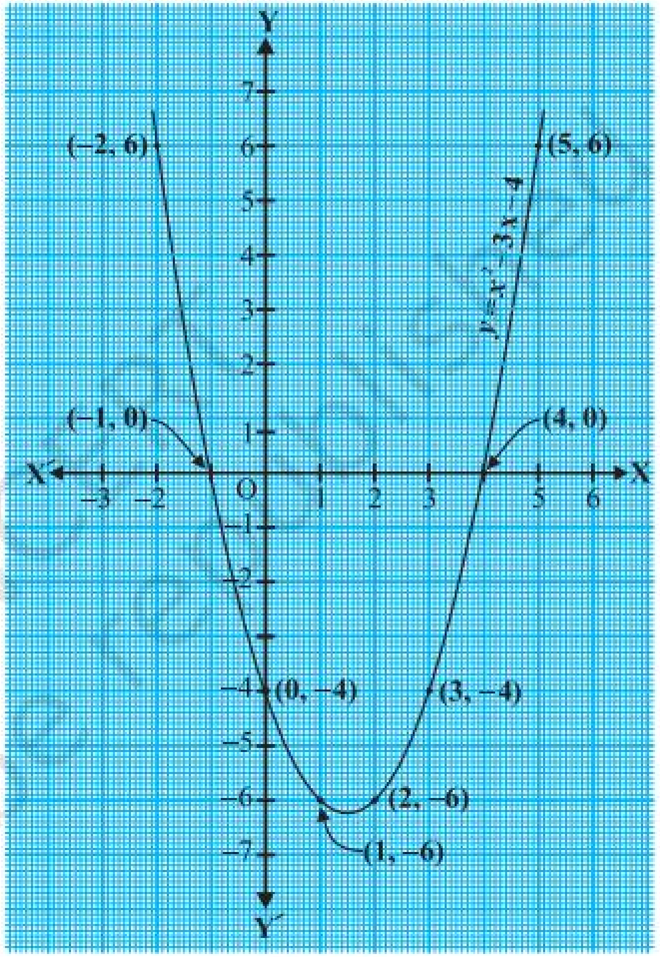


Fig. 2.2

2.	The value of k for which the system of equations 3x-ky= 7 and 6x+ 10y =3 is inconsistent, is A) -10 B) -5 C) 5 D) 7	1

We can now summarise the behaviour of lines representing a pair of linear equations in two variables and the existence of solutions as follows:

(i) the lines may intersect in a single point. In this case, the pair of equations has a unique solution (consistent pair of equations).

(ii) the lines may be parallel. In this case, the equations have no solution (inconsistent pair of equations).

(iii) the lines may be coincident. In this case, the equations have infinitely many solutions [dependent (consistent) pair of equations].

Table 3.1

Sl No.	Pair of lines	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the ratios	Graphical representation	Algebraic interpretation
1.	$x - 2y = 0$ $3x + 4y - 20 = 0$	$\frac{1}{3}$	$\frac{-2}{4}$	$\frac{0}{-20}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution (unique)
2.	$2x + 3y - 9 = 0$ $4x + 6y - 18 = 0$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{-9}{-18}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions
3.	$x + 2y - 4 = 0$ $2x + 4y - 12 = 0$	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{-4}{-12}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution

3.	Which of the following statements is **not** true? A) A number of secants can be drawn at any point on the circle. B) Only one tangent can be drawn at any point on a circle. C) A chord is a line segment joining two points on the circle D) From a point inside a circle only two tangents can be drawn.	1

So, let us consider a circle and a line PQ. There can be three possibilities given in Fig. 10.1 below:

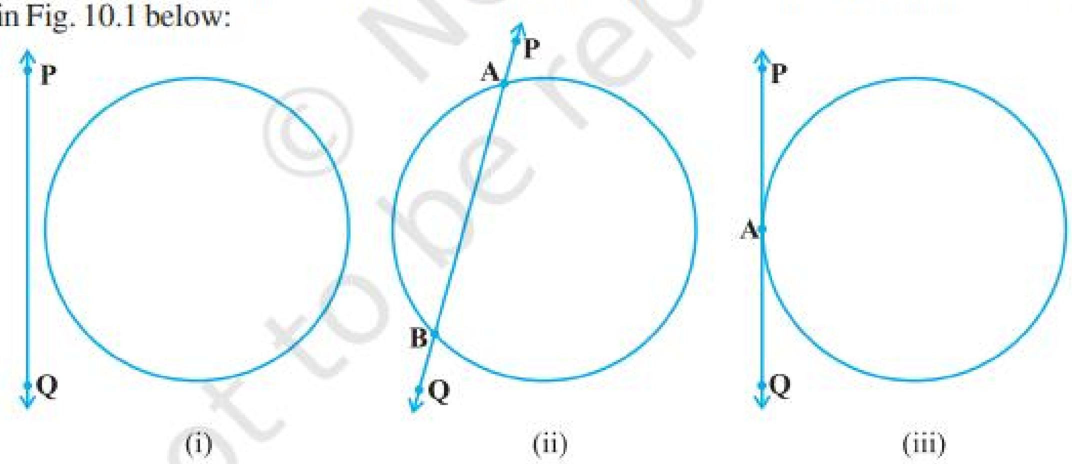


Fig. 10.1

In Fig. 10.1 (i), the line PQ and the circle have no common point. In this case, PQ is called a **non-intersecting** line with respect to the circle. In Fig. 10.1 (ii), there are two common points A and B that the line PQ and the circle have. In this case, we call the line PQ a **secant** of the circle. In Fig. 10.1 (iii), there is only one point A which is common to the line PQ and the circle. In this case, the line is called a **tangent** to the circle.

In various positions, the wire intersects the circular wire at P and at another point Q_1 or Q_2 or Q_3, etc. In one position, you will see that it will intersect the circle at the point P only (see position A′B′ of AB). This shows that a tangent exists at the point P of the circle. On rotating further, you can observe that in all other positions of AB, it will intersect the circle at P and at another point, say R_1 or R_2 or R_3, etc. So, you can observe that **there is only one tangent at a point of the circle**.

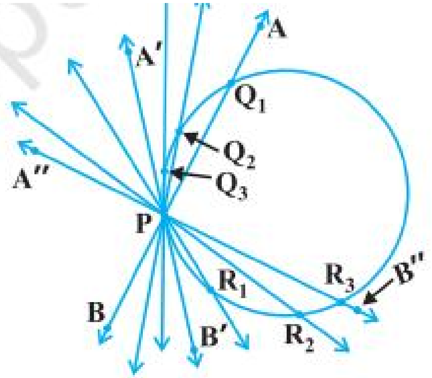


Fig. 10.3 (i)

While doing activity above, you must have observed that as the position AB moves towards the position A′ B′, the common point, say Q_1, of the line AB and the circle gradually comes nearer and nearer to the common point P. Ultimately, it coincides with the point P in the position A′B′ of A″B″. Again note, what happens if ‘AB’ is rotated rightwards about P? The common point R_3 gradually comes nearer and nearer to P and ultimately coincides with P. So, what we see is:

The tangent to a circle is a special case of the secant, when the two end points of its corresponding chord coincide.

Activity 3 : Draw a circle on a paper. Take a point P inside it. Can you draw a tangent to the circle through this point? You will find that all the lines through this point intersect the circle in two points. So, it is not possible to draw any tangent to a circle through a point inside it [see Fig. 10.6 (i)].

Next take a point P on the circle and draw tangents through this point. You have already observed that there is only one tangent to the circle at such a point [see Fig. 10.6 (ii)].

Finally, take a point P outside the circle and try to draw tangents to the circle from this point. What do you observe? You will find that you can draw exactly two tangents to the circle through this point [see Fig. 10.6 (iii)].

We can summarise these facts as follows:

Case 1 : There is no tangent to a circle passing through a point lying inside the circle.

Case 2 : There is one and only one tangent to a circle passing through a point lying on the circle.

Case 3 : There are exactly two tangents to a circle through a point lying outside the circle.

In Fig. 10.6 (iii), T_1 and T_2 are the points of contact of the tangents PT_1 and PT_2 respectively.

The length of the segment of the tangent from the external point P and the point of contact with the circle is called the **length of the tangent**

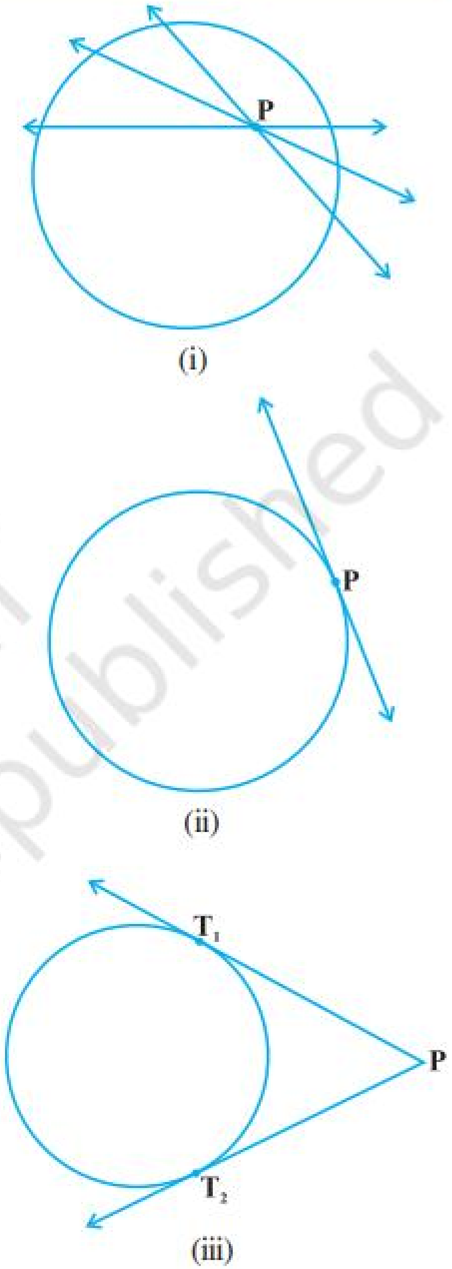
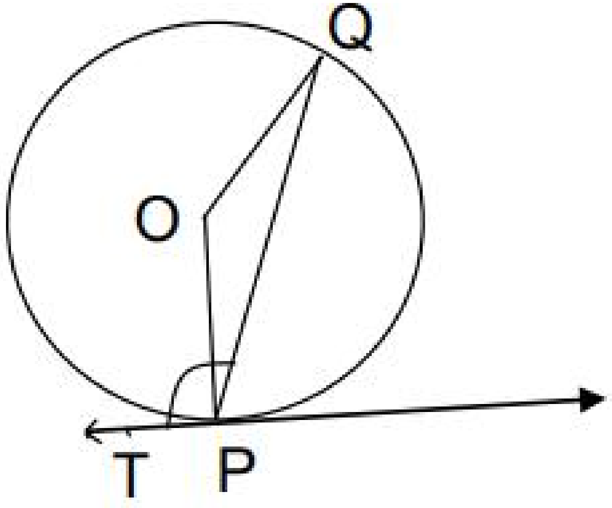


Fig. 10.6

4.	If nth term of an A.P. is 7n-4 then the common difference of the A.P. is A) 7 B) 7n C) - 4 D) 4	1

5.	The radius of the base of a right circular cone and the radius of a sphere are each 5 cm in length. If the volume of the cone is equal to the volume of the sphere then the height of the cone is A) 5 cm B) 20 cm C) 10 cm D) 4 cm	1

6.	If $\tan\theta = \frac{5}{2}$ then $\frac{4\,sin\theta + cos\theta}{4\,sin\theta\ - cos\theta}$ is equal to A) $\frac{11}{9}$ B) $\frac{3}{2}$ C) $\frac{9}{11}$ D) 4	1

7.	In the given figure, a tangent has been drawn at a point P on the circle centred at O.  If ∠ TPQ= 110^{O} then $\angle POQ$ is equal to A) 110^{O} B) 70^{O} C) 140^{O} D)55^{O}	1

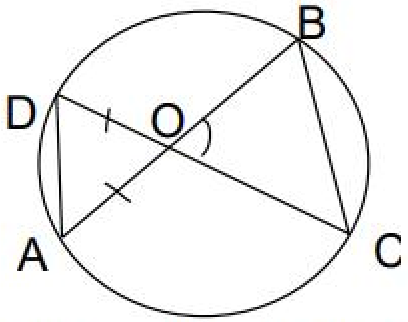
8.	A quadratic polynomial having zeroes - $\sqrt{\frac{5}{2}}$ and $\sqrt{\frac{5}{2}}$ is A) $x^2 - 5\sqrt{2}$ x +1 B) $8x^2$ - 20 C) 15x^2- 6 D) x^2 - 2$\sqrt{5}$ x -1	1

9. Consider the frequency distribution of 45 observations. — 1

Class	0-10	10-20	20-30	30-40	40-50
Frequency	5	9	15	10	6

The upper limit of median class is

A) 20 B) 10 C) 30 D) 40

10.	O is the point of intersection of two chords AB and CD of a circle.  If $\angle BOC = 80^{O}$ and OA = OD then ΔODA *and* ΔOBC are A) equilateral and similar B) isosceles and similar C) isosceles but not similar D) not similar	1

11.	The roots of the quadratic equation x^2+x-1 = 0 are A) Irrational and distinct B) not real C) rational and distinct D) real and equal	1

Since $b^2 - 4ac$ determines whether the quadratic equation $ax^2 + bx + c = 0$ has real roots or not, $b^2 - 4ac$ is called the **discriminant** of this quadratic equation.

So, a quadratic equation $ax^2 + bx + c = 0$ has

(i) **two distinct real roots, if $b^2 - 4ac > 0$,**

(ii) **two equal real roots, if $b^2 - 4ac = 0$,**

(iii) **no real roots, if $b^2 - 4ac < 0$.**

Let us consider some examples.

Example 7: Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$, and hence find the nature of its roots.

Solution : The given equation is of the form $ax^2 + bx + c = 0$, where $a = 2$, $b = -4$ and $c = 3$. Therefore, the discriminant

$$b^2 - 4ac = (-4)^2 - (4 \times 2 \times 3) = 16 - 24 = -8 < 0$$

So, the given equation has no real roots.

12.	If $\theta = 30^o$ then the value of 3tanθ is A)1 B) $\frac{1}{\sqrt{3}}$ C) $\frac{3}{\sqrt{3}}$ (D) not defined	1

$\angle$ **A**	**0°**	**30°**	**45°**	**60°**	**90°**
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

13.	The volume of a solid hemisphere is $\frac{396}{7}\,cm^3$.The total surface area of the solid hemisphere (in sq.cm) is A) $\frac{396}{7}$ B) $\frac{594}{7}$ C) $\frac{549}{7}$ D) $\frac{604}{7}$	1

14.	In a bag containing 24 balls, 4 are blue, 11 are green and the rest are white. One ball is drawn at random. The probability that drawn ball is white in colour is $A)\frac{1}{6}$ B) $\frac{3}{8}$ C) $\frac{11}{24}$ D) $\frac{5}{8}$	1

15.	The point on the x- axis nearest to the point (-4,-5) is A) (0, 0) B) (-4, 0) C) (-5, 0) D) ($\sqrt{41}$, 0)	1

16.	Which of the following gives the middle most observation of the data? A) Median B) Mean C) Range D) Mode	1

Median: The middle value in a data set when the values are arranged in order.
Mean: The average of the data set.
Range: The difference between the largest and smallest values in the data set.
Mode: The most frequently occurring value in the data set.

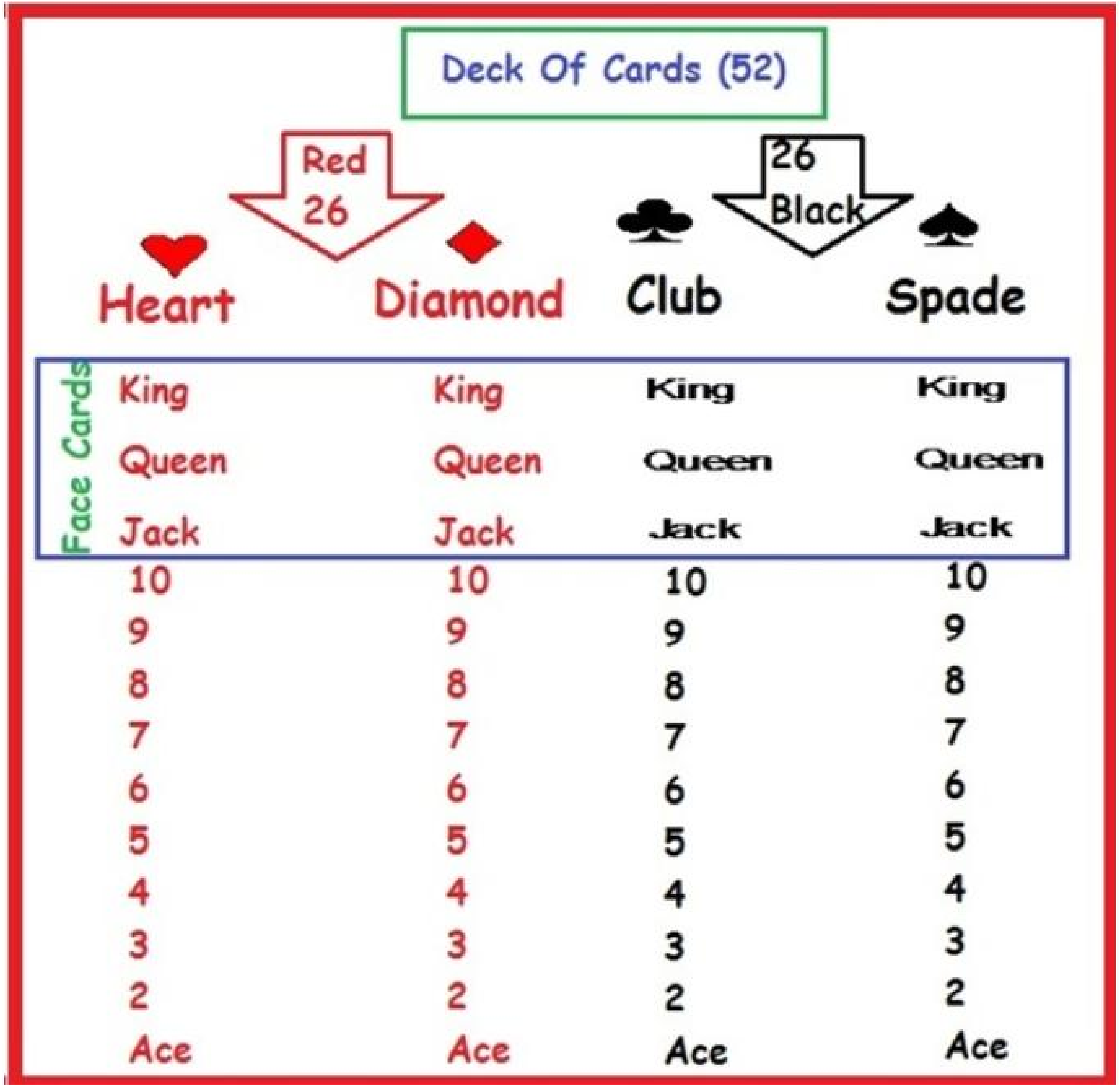
17.	A point on the x-axis divides the line segment joining the points A(2, -3) and B(5, 6) in the ratio 1:2. The point is A) (4, 0) B) $\left(\frac{7}{2}, \frac{3}{2}\right)$ C) (3, 0) D) (0,3)	1

So, the coordinates of the point P(x, y) which divides the line segment joining the points A(x_1, y_1) and B(x_2, y_2), internally, in the ratio $m_1 : m_2$ are

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right) \quad (2)$$

This is known as the **section formula**.

18.	A card is drawn from a well shuffled deck of playing cards. The probability of getting red face card is $A)\ \frac{3}{13}$ B) $\frac{1}{2}$ C) $\frac{3}{52}$ D) $\frac{3}{26}$	1



Deck Of Cards (52)
Red 26
26 Black
Heart
Diamond
Club
Spade
Face Cards
King
Queen
Jack
10
9
8
7
6
5
4
3
2
Ace

	DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option A)Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A) B)Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A) C)Assertion (A) is true but reason (R) is false. D)Assertion (A) is false but reason (R) is true.	
19.	Assertion (A): HCF of any two consecutive even natural numbers is always 2. Reason (R): Even natural numbers are divisible by 2.	1

	DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option A)Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A) B)Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A) C)Assertion (A) is true but reason (R) is false. D)Assertion (A) is false but reason (R) is true.	
20.	Assertion (A): If the radius of sector of a circle is reduced to its half and angle is doubled then the perimeter of the sector remains the same.	1
	Reason (R): The length of the arc subtending angle θ at the centre of a circle of radius r $= \frac{\Pi r\theta}{180}$.	

	Section B	
	Section B consists of 5 questions of 2 marks each.	
21.	(A)Find the H.C.F and L.C.M of 480 and 720 using the Prime factorisation method.	2

21. (A)	$480 = 2^5 \times 3 \times 5$ $720 = 2^4 \times 3^2 \times 5$ LCM (480,720) = $2^5 \times 3^2 \times 5 = 1440$ HCF (480, 720) = $2^4 \times 3 \times 5 = 240$	½ ½ ½ ½

	OR (A) The H.C.F of 85 and 238 is expressible in the form 85m -238. Find the value of m.		

(B)	85 = 5x17, 238 = 2x7x17 HCF(85, 238) = 17 17 = 85xm -238 m = 3	1 1	

22.	(A) Two dice are rolled together bearing numbers 4, 6, 7, 9, 11, 12. Find the probability that the product of numbers obtained is an odd number	2

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[(4, 4), (4, 6), (4, 7), (4, 9), (4, 11), (4, 12),
 (6, 4), (6, 6), (6, 7), (6, 9), (6, 11), (6, 12),
 (7, 4), (7, 6), (7, 7), (7, 9), (7, 11), (7, 12),
 (9, 4), (9, 6), (9, 7), (9, 9), (9, 11), (9, 12),
 (11, 4), (11, 6), (11, 7), (11, 9), (11, 11), (11, 12),
 (12, 4), (12, 6), (12, 7), (12, 9), (12, 11), (12, 12)]
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Total number of possible outcomes = 6x6=36

For a product to be odd, both the numbers should be odd.

Favourable outcomes are (7,7) (7,9) (7,11) (9,7) (9,9) (9, 11) (11,7) (11,9) (11,11)

no. of favourable outcomes = 9

P (product is odd) = $\frac{9}{36}$ or $\frac{1}{4}$

Sample space when two dice are thrown simultaneously

$$\begin{bmatrix} (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6) \\ (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) \\ (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6) \\ (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6) \\ (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6) \\ (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \end{bmatrix}$$

	OR (B) How many positive three digit integers have the hundredths digit 8 and unit's digit 5? Find the probability of selecting one such number out of all three digit numbers.		

Total number of three-digit numbers = 900.
Numbers with hundredth digit 8 & and unit's digit 5 are 805,815, 825,....,895
Number of favourable outcomes = 10
P(selecting one such number) = $\frac{10}{900}$ or $\frac{1}{90}$

23.	Evaluate: $\frac{2\sin^2 60^o - \tan^2 30^o}{\sec^2 45^o}$	2

$$\frac{2\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2}{\left(\sqrt{2}\right)^2}$$

$$= \frac{7}{12}$$

$\angle$ A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

24.	Find the point(s) on the x-axis which is at a distance of $\sqrt{41}$ units from the point (8, -5).	2

Let the required point be (x,0)

$\sqrt{(8-x)^2 + 25} = \sqrt{41}$

=> $(8-x)^2 = 16$

=> $8 - x = \pm 4$

=> $x = 4 , 12$

Two points on the x-axis are (4,0) & (12,0).

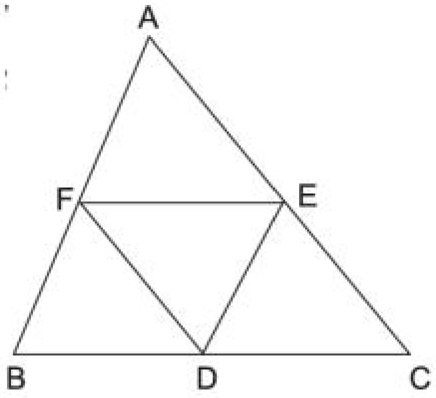
25.	Show that the points A(-5,6), B(3, 0) and C(9, 8) are the vertices of an isosceles triangle.	2

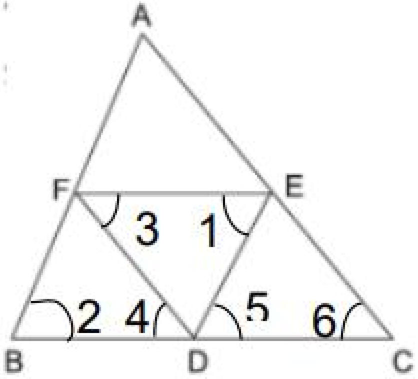
AB = $\sqrt{(3+5)^2 + (0-6)^2}$ = 10

BC = $\sqrt{(9-3)^2 + (8-0)^2}$ = 10

AC = $\sqrt{(9+5)^2 + (8-6)^2}$ = $10\sqrt{2}$

Since AB = BC, therefore Δ ABC is isosceles

	Section C	
	Section C consists of 6 questions of 3 marks each.	
26.	(A) In ΔABC, D, E and F are midpoints of BC,CA and AB respectively. Prove that $\triangle FBD \sim \triangle DEF$ and $\triangle DEF \sim \triangle ABC$ 	3



Since D, E, F are the mid points of BC, CA, AB respectively

Therefore, EF||BC, DF||AC, DE||AB

BDEF is a parallelogram

∠ 1= ∠ 2 & ∠ 3 = ∠ 4

Δ FBD ~ Δ DEF

Also, DCEF is a parallelogram

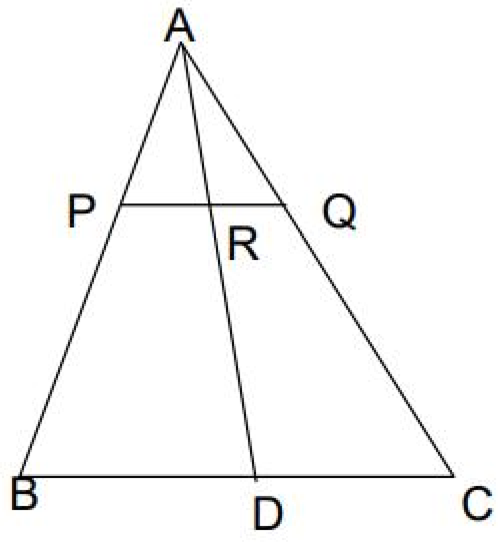
∠ 3= ∠ 6 & ∠ 1 = ∠ 2 (proved above)

Δ DEF ~ Δ ABC

OR

(B) In ΔABC, P and Q are points on AB and AC respectively such that PQ is parallel to BC.

Prove that the median AD drawn from A on BC bisects PQ.



Since PQ//BC therefore Δ APR ~ Δ ABD

=> $\frac{AP}{AB} = \frac{PR}{BD}$ …….. (i)

Δ AQR ~ Δ ACD

=> $\frac{AQ}{AC} = \frac{RQ}{DC}$ …….. (ii)

Now, $\frac{AP}{AB} = \frac{AQ}{AC}$ ……….(iii)

Using (i), (ii) & (iii), $\frac{PR}{BD} = \frac{RQ}{DC}$

But, BD = DC

=> PR = RQ or AD bisects PQ

27.	The sum of two numbers is 18 and the sum of their reciprocals is 9/40. Find the numbers.	3

Let the numbers be x and 18-x.

$\frac{1}{x} + \frac{1}{18-x} = \frac{9}{40}$

=> $18 \times 40 = 9x(18-x)$

=> $x^2 - 18x + 80 = 0$

=> $(x-10)(x-8) = 0$

=> $x = 10, 8$.

=> $18 - x = 8, 10$

Hence two numbers are 8 and 10.

28.	If α and β are zeroes of a polynomial $6x^2$ -5x+1 then form a quadratic polynomial whose zeroes are α^2 and β^2.	3

From given polynomial $\alpha + \beta = \frac{5}{6}$, $\alpha\beta = \frac{1}{6}$

$\alpha^2 + \beta^2 = (\frac{5}{6})^2 - 2 \times \frac{1}{6} = \frac{13}{36}$

And $\alpha^2 \beta^2 = (\frac{1}{6})^2 = \frac{1}{36}$

$x^2 - \frac{13}{36}x + \frac{1}{36}$

⇨ Required polynomial is $36x^2$ -13 x+1

29.	If cosθ + sinθ = 1 , then prove that cosθ - sinθ = ±1	3

$(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2 = 2(\cos^2\theta + \sin^2\theta) = 2$

=> $(1)^2 + (\cos\theta - \sin\theta)^2 = 2$

=> $(\cos\theta - \sin\theta)^2 = 1$

=> $\cos\theta - \sin\theta = \pm 1$

30.	(A) The minute hand of a wall clock is 18 cm long. Find the area of the face of the clock described by the minute hand in 35 minutes.	3

Angle described by minute hand in 5 min = 30°.

length of minute hand =18 cm = r.

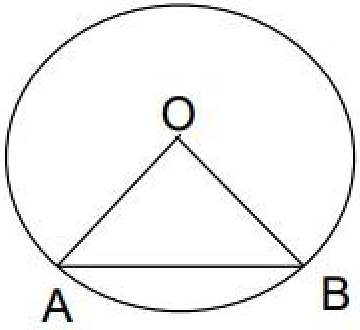
Area swept by minute hand in 35 minutes

$=\left(\frac{22}{7}\times 18\times 18\times\frac{30}{360}\right)\times 7$

$= 594\ cm^2$.

OR

(B) AB is a chord of a circle centred at O such that $\angle AOB = 60^\circ$. If OA = 14 cm then find the area of the minor segment. (take $\sqrt{3}$ =1.73)



Area of minor segment = Ar. Sector OAB- Ar. Δ OAB

$$= \frac{60}{360} \times \frac{22}{7} \times 14 \times 14 - \frac{\sqrt{3}}{4} \times 14 \times 14$$

$$= 17.89\ cm^2$$

31.	Prove that $\sqrt{3}$ is an irrational number.	3

Let $\sqrt{3}$ be a rational number.

$\therefore \sqrt{3} = \frac{p}{q}$, where $q \neq 0$ and let p & q be co-prime.

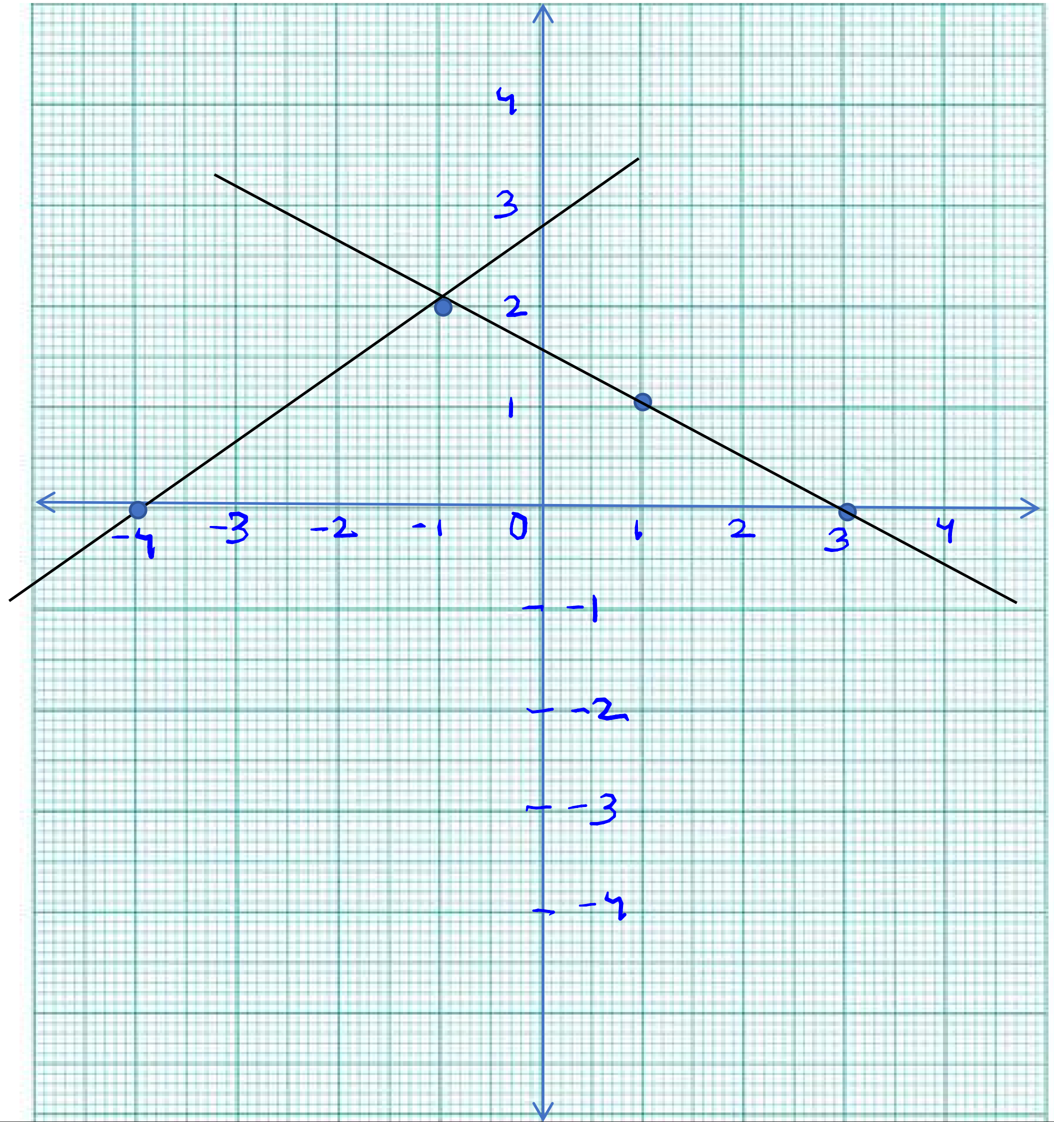
$3q^2 = p^2 \Rightarrow p^2$ is divisible by 3 $\Rightarrow$ p is divisible by 3 ----- (i)

$\Rightarrow p = 3a$, where 'a' is some integer

$9a^2 = 3q^2 \Rightarrow q^2 = 3a^2 \Rightarrow q^2$ is divisible by 3 $\Rightarrow$ q is divisible by 3----- (ii)

(i) and (ii) leads to contradiction as 'p' and 'q' are co-prime.

	Section D	
	Section D consists of 4 questions of 5 marks each	
32.	(A) Solve the following system of linear equations graphically: $x+2y = 3$, $2x-3y+8 = 0$	5



x	3	1
y	0	1

x	-4	-1
y	0	2

$x+2y=3$, $2x-3y+8=0$

Correct graph of each equation

Solution $x=-1$ and $y=2$

OR

(B) Places A and B are 180 km apart on a highway. One car starts from A and another from B at the same time. If the car travels in the same direction at different speeds, they meet in 9 hours. If they travel towards each other with the same speeds as before, they meet in an hour. What are the speeds of the two cars?

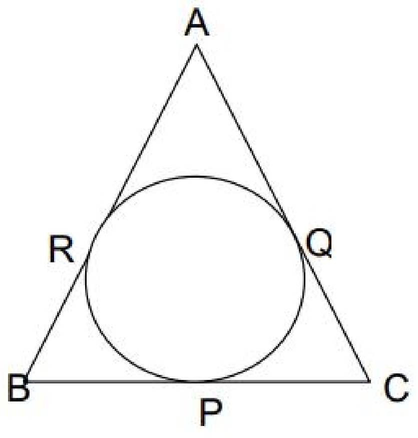
Let car I starts from A with speed x km/hr and car II Starts from B with speed y km/hr (x>y)

Case I- when cars are moving in the same direction.
Distance covered by car I in 9 hours = 9x.
Distance covered by car II in 9 hours = 9y
Therefore 9 (x-y) = 180
=> x-y= 20 .... (i)

case II- when cars are moving in opposite directions.

Distance covered by Car I in 1 hour = x
Distance covered by Car II in 1 hour = y

Therefore x + y=180 .... (ii)
Solving (i) and (ii) we get, x=100 km/hr, y=80 km/hr.

33.	Prove that the lengths of tangents drawn from an external point to a circle are equal. Using above result, find the length BC of ΔABC. Given that, a circle is inscribed in ΔABC touching the sides AB, BC and CA at R, P and Q respectively and AB= 10 cm, AQ= 7cm ,CQ= 5cm.  	5

Theorem 10.2 : *The lengths of tangents drawn from an external point to a circle are equal.*

Proof : We are given a circle with centre O, a point P lying outside the circle and two tangents PQ, PR on the circle from P (see Fig. 10.7). We are required to prove that PQ = PR.

For this, we join OP, OQ and OR. Then $\angle$ OQP and $\angle$ ORP are right angles, because these are angles between the radii and tangents, and according to Theorem 10.1 they are right angles. Now in right triangles OQP and ORP,

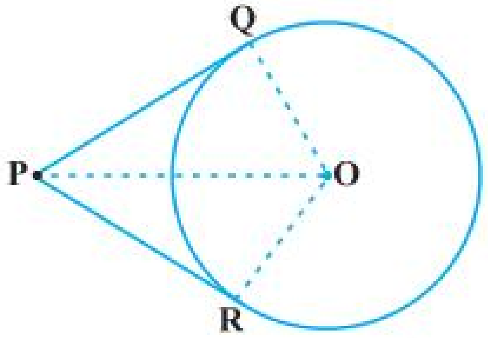


Fig. 10.7

OQ = OR (Radii of the same circle)

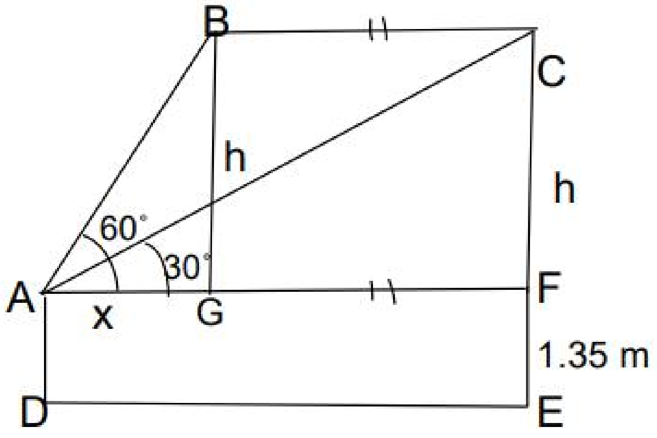
OP = OP (Common)

Therefore, Δ OQP $\cong$ Δ ORP (RHS)

This gives PQ = PR (CPCT)

AR = AQ = 7cm
BP = BR = AB-AR = 3cm
CP =CQ = 5cm
BC = BP+PC = 3+5 = 8 cm

34.	A boy whose eye level is 1.35 m from the ground, spots a balloon moving with the wind in a horizontal line at some height from the ground. The angle of elevation of the balloon from the eyes of the boy at an instant is 60^o. After 12 seconds, the angle of elevation reduces to 30°. If the speed of the wind is 3m/s then find the height of the balloon from the ground. (Use $\sqrt{3}$= 1.73)	5



Let A be the eye level & B, C are positions of balloon
Distance covered by balloon in 12 sec = 3x12 = 36 m
BC = GF = 36 m

tan 60^0 = $\sqrt{3}$ = $\frac{h}{x}$
=> h = $x\sqrt{3}$ .... (i)

tan 30^0 = $\frac{1}{\sqrt{3}}$ = $\frac{h}{x+36}$
=> h = $\frac{x+36}{\sqrt{3}}$ .... (ii)
Solving (i) and (ii) h= $18\sqrt{3}$ = 31.14 m
Height of balloon from ground = 1.35 + 31.14 = 32.49 m

35.	Find the mean and median of the following data:	5

Class	85-90	90-95	95-100	100-105	105-110	110-115
frequency	15	22	20	18	20	25

Class	x	f	$u = \frac{x-102.5}{5}$	fu	cf
85-90	87.5	15	-3	-45	15
90-95	92.5	22	-2	-44	37
95-100	97.5	20	-1	-20	57
100-105	102.5	18	0	0	75
105-110	107.5	20	1	20	95
110-115	112.5	25	2	50	120
		$\Sigma f = 120$		$\Sigma fu = -39$	

Mean = $\overline{x}$ = 102.5 - 5 x $\frac{39}{120}$

= 100.875

Median class is 100-105

Median = 100 + $\frac{5}{18}$ (60-57) = 100.83

Percentage of female teachers	Number of states/U.T. (f_i)	x_i	$d_i = x_i - 50$	$u_i = \frac{x_i - 50}{10}$	$f_i x_i$	$f_i d_i$	$f_i u_i$
15 - 25	6	20	–30	–3	120	–180	–18
25 - 35	11	30	–20	–2	330	–220	–22
35 - 45	7	40	–10	–1	280	–70	–7
45 - 55	4	50	0	0	200	0	0
55 - 65	4	60	10	1	240	40	4
65 - 75	2	70	20	2	140	40	4
75 - 85	1	80	30	3	80	30	3
Total	**35**				**1390**	**–360**	**–36**

From the table above, we obtain $\Sigma f_i = 35$, $\Sigma f_i x_i = 1390$,

$$\Sigma f_i d_i = -360, \quad \Sigma f_i u_i = -36.$$

Using the direct method, $\bar{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{1390}{35} = 39.71$

Using the assumed mean method,

$$\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 50 + \frac{(-360)}{35} = 39.71$$

Using the step-deviation method,

$$\bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h = 50 + \left(\frac{-36}{35}\right) \times 10 = 39.71$$

Therefore, the mean percentage of female teachers in the primary schools of rural areas is 39.71.

After finding the median class, we use the following formula for calculating the median.

$$\text{Median} = l + \left(\frac{\frac{n}{2} - \text{cf}}{f}\right) \times h,$$

where l = lower limit of median class,

n = number of observations,

cf = cumulative frequency of class preceding the median class,

f = frequency of median class,

h = class size (assuming class size to be equal).

In a grouped frequency distribution, it is not possible to determine the mode by looking at the frequencies. Here, we can only locate a class with the maximum frequency, called the **modal class**. The mode is a value inside the modal class, and is given by the formula:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

where l = lower limit of the modal class,

h = size of the class interval (assuming all class sizes to be equal),

f_1 = frequency of the modal class,

f_0 = frequency of the class preceding the modal class,

f_2 = frequency of the class succeeding the modal class.

Let us consider the following examples to illustrate the use of this formula.

Example 5 : A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a household:

Family size	1 - 3	3 - 5	5 - 7	7 - 9	9 - 11
Number of families	7	8	2	2	1

Find the mode of this data.

Solution : Here the maximum class frequency is 8, and the class corresponding to this frequency is 3 – 5. So, the modal class is 3 – 5.

Now

modal class = 3 – 5, lower limit (l) of modal class = 3, class size (h) = 2

frequency (f_1) of the modal class = 8,

frequency (f_0) of class preceding the modal class = 7,

frequency (f_2) of class succeeding the modal class = 2.

Now, let us substitute these values in the formula :

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 3 + \left(\frac{8 - 7}{2 \times 8 - 7 - 2}\right) \times 2 = 3 + \frac{2}{7} = 3.286$$

Therefore, the mode of the data above is 3.286.

OR

The monthly expenditure on milk in 200 families of a Housing Society is given below

Monthly Expenditure (in Rs.)	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
Number of families	24	40	33	x	30	22	16	7

Find the value of x and also find the mean expenditure

Monthly Expenditure	f_i	x_i	f_ix_i
1000-1500	24	1250	30,000
1500-2000	40	1750	70,000
2000-2500	33	2250	74,250
2500-3000	**X=28**	**2750**	**77,000**
3000-3500	30	3250	97,500
3500-4000	22	3750	82,500
4000-4500	16	4250	68,000
4500-5000	7	4750	33,250

172+x=200

X=28

Mean= $\frac{532500}{200}$

= 2662.5

Example 8 : The median of the following data is 525. Find the values of x and y, if the total frequency is 100.

Class intervals	Frequency
0 - 100	2
100 - 200	5
200 - 300	x
300 - 400	12
400 - 500	17
500 - 600	20
600 - 700	y
700 - 800	9
800 - 900	7
900 - 1000	4

Class intervals	Frequency	Cumulative frequency
0 - 100	2	2
100 - 200	5	7
200 - 300	x	$7 + x$
300 - 400	12	$19 + x$
400 - 500	17	$36 + x$
500 - 600	20	$56 + x$
600 - 700	y	$56 + x + y$
700 - 800	9	$65 + x + y$
800 - 900	7	$72 + x + y$
900 - 1000	4	$76 + x + y$

It is given that $n = 100$

So, $76 + x + y = 100$, i.e., $x + y = 24$ (1)

The median is 525, which lies in the class 500 – 600

So, $l = 500$, $f = 20$, $\text{cf} = 36 + x$, $h = 100$

Using the formula : $$\text{Median} = l + \left(\frac{\frac{n}{2} - \text{cf}}{f}\right)h,$$ we get

$$525 = 500 + \left(\frac{50 - 36 - x}{20}\right) \times 100$$

i.e., $525 - 500 = (14 - x) \times 5$

i.e., $25 = 70 - 5x$

i.e., $5x = 70 - 25 = 45$

So, $x = 9$

Therefore, from (1), we get $9 + y = 24$

i.e., $y = 15$

	Section E
	Section E consists of 3 case study based questions of 4 marks each.
36.	Ms. Sheela visited a store near her house and found that the glass jars are arranged one above the other in a specific pattern. On the top layer there are 3 jars. In the next layer there are 6 jars. In the 3rd layer from the top there are 9 jars and so on till the 8th layer. On the basis of the above situation answer the following questions. (i) Write an A.P whose terms represent the number of jars in different layers starting from top . Also, find the common difference. (ii) Is it possible to arrange 34 jars in a layer if this pattern is continued? Justify your answer. (iii) (A) If there are 'n' number of rows in a layer then find the expression for finding the total number of jars in terms of n. Hence find S_8 . **OR** (iii) (B) The shopkeeper added 3 jars in each layer. How many jars are there in the 5th layer from the top?

36.(i) First term a = 3, A.P is 3, 6, 9, 12.....,24

common difference d = 6-3 = 3

(ii) 34 = 3+ (n-1)3

=> n = 34/3 = $11\frac{1}{3}$ which is not a positive integer.

Therefore, it is not possible to have 34 jars in a layer if the given pattern is continued.

(iii)(A)

$S_n = \frac{n}{2}$ [2x3 + (n-1) 3]

$= \frac{n}{2}$ [6 + 3n-3]

$= \frac{n}{2}$ [3+3n]

$= 3\,\frac{n}{2}$ [1+n]

s_8 = 3 x $\frac{8}{2}$ (1+8)

= 108

OR

(iii) (B) A.P will be 6, 9, 12, ...

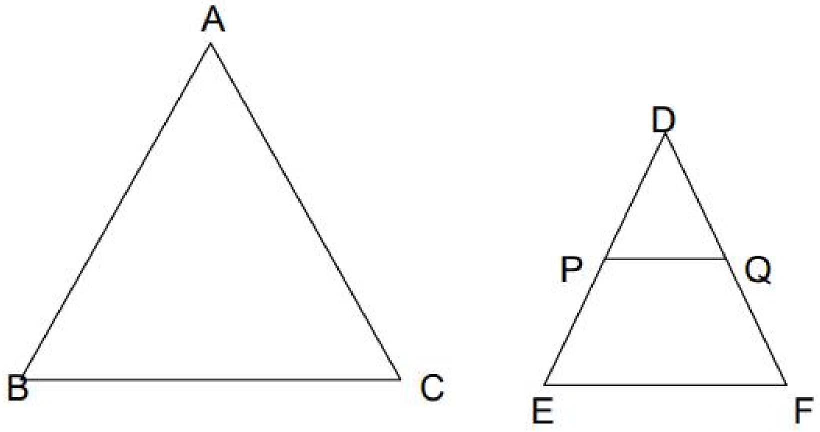
a= 6, d=3

t_5 = 6 + (5-1)3

= 6 + 12

= 18

37.



Triangle is a very popular shape used in interior designing. The picture given above shows a cabinet designed by a famous interior designer.

Here the largest triangle is represented by △ ABC and smallest one with shelf is represented by △ DEF. PQ is parallel to EF.

(i) Show that △ DPQ ~ △ DEF.

(ii) If DP= 50 cm and PE = 70 cm then find $\frac{PQ}{EF}$.

(iii) (A) If 2AB = 5DE and △ ABC ~ △ DEF then show that $\frac{perimeter\ of\ \triangle ABC}{perimeter\ of\ \triangle DEF}$ is constant.

OR

(iii) (B) If AM and DN are medians of triangles ABC and DEF respectively then prove that △ ABM ~ △ DEN.

(i) ∠DPQ = ∠DEF

∠PDQ = ∠EDF

Therefore Δ DPQ ~ Δ DEF

(ii) DE = 50 + 70 = 120 cm

$\frac{DP}{DE} = \frac{PQ}{EF}$

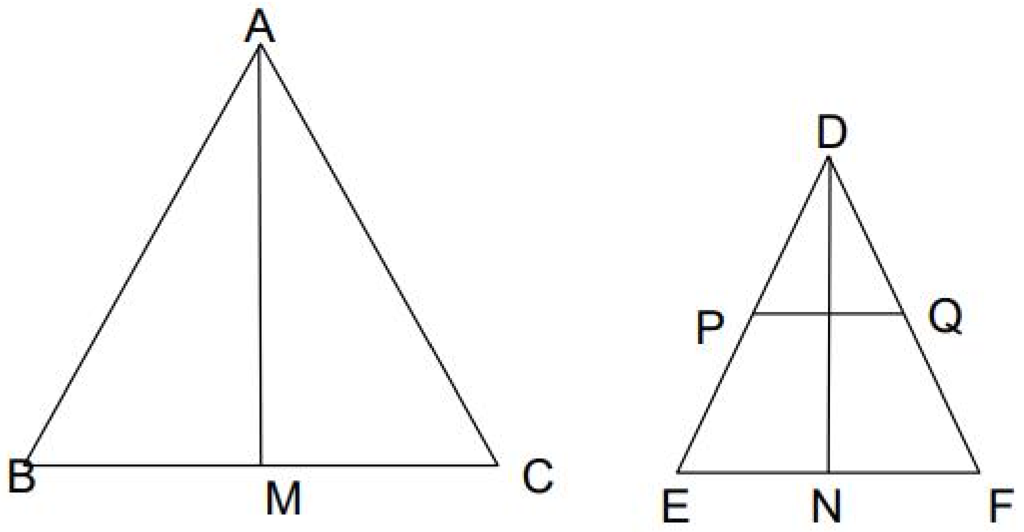
Therefore $\frac{PQ}{EF} = \frac{50}{120}$ or $\frac{5}{12}$

(iii) (A)

$\frac{AB}{DE} = \frac{5}{2} = \frac{BC}{EF} = \frac{AC}{DF}$

⇒ AB = $\frac{5}{2}$ DE

$\frac{perimeter\ of\ \triangle ABC}{perimeter\ of\ \triangle DEF} = \frac{\frac{5}{2}(DE + EF + FD)}{DE + EF + FD} = \frac{5}{2}$ (Constant) (iii)(B)



$\frac{AB}{DE} = \frac{BC}{EF} = \frac{BC/2}{EF/2} = \frac{BM}{EN}$

Also $\angle B = \angle E$

Therefore $\triangle ABM \sim \triangle DEN$.

38.

Metallic silos are used by farmers for storing grains. Farmer Girdhar has decided to build a new metallic silo to store his harvested grains. It is in the shape of a cylinder mounted by a cone.

Dimensions of the conical part of a silo is as follows:

Radius of base = 1.5 m

Height = 2 m

Dimensions of the cylindrical part of a silo is as follows:

Radius = 1.5 m

Height = 7 m

On the basis of the above information answer the following questions.

(i) Calculate the slant height of the conical part of one silo.

(ii) Find the curved surface area of the conical part of one silo. 1

(iii)(A) Find the cost of metal sheet used to make the curved cylindrical part of 1 silo at the rate of ₹2000 per m^2. 2

OR

(iii) (B) Find the total capacity of one silo to store grains. 2

38. (i)

l= $\sqrt{r^2 + h^2}$

= $\sqrt{(1.5)^2 + (2)^2}$

= $\sqrt{2.25 + 4}$

= $\sqrt{6.25}$

= 2.5 m

(ii)

CSA of cone = Πrl

= $\frac{22}{7}$ x 1.5 x 2.5

= 11.78 m^2

(iii) (A)

CSA of cylinder = 2Π rh

= 2 x $\frac{22}{7}$ x 1.5 x 7

= 66 m^2

Cost of metal sheet used = 66 x 2000

= ₹1,32,000

OR

(iii) (B)

Volume of cylinder = $\Pi\, r^2$ h

= $\frac{22}{7}$ x $(1.5)^2$ x 7

= 49.5 m^3

Volume of cone = $\frac{1}{3}\Pi\, r^2$ h

=$\frac{1}{3}$ x $\frac{22}{7}$ x $(1.5)^2$ x 2

= 4.71 m^3

Total capacity = 49.5 + 4.71 = 54.21 m^3