Class-X

Mathematics Standard (041)

SEC - A

1. (b) infinite

2. (d) $ab = 6$

3. ~~(a) 7~~ (b) ~~3~~

4. (d) 10

5. (a) $x^2 - 4x + 1 = 0$

6. (a) $-\frac{17}{7}$

7. (d) 3 units

8. (c) 6.5 cm

9. (c) 8000 m^3

10. (b) 21

11. (a) 3 cm

12. (a) sin 60°

13. (b) $\angle B = \angle D$

14. (c) −32

15. (c) $\frac{3}{4}$

16. (a) 30°

17. (b) $\frac{7}{0.01}$

18. (a) decreases by 2

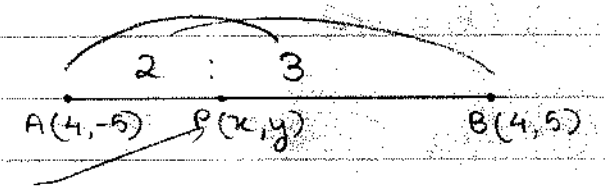
19. (c) Assertion (A) is true, but Reason (R) is false.

20. (c) Assertion (A) is true, but Reason (R) is false.
[only $\boxed{1 \text{ prime factor} = 5}$ ∵ prime factorisation of a prime number is the number itself]

SEC-B

21. A(4, −5) and B(4, 5)

(a) Let the coordinates of point P which divides AB such that AP : AB = 2 : 5 be P(x, y).



$$\frac{AP}{AB} = \frac{2}{5}$$

$$\Rightarrow \frac{AP + PB}{AP} = \frac{5}{2}$$

$$\Rightarrow 2PB = 3AP \Rightarrow AP : PB = 2 : 3$$

P divides AB internally in the ratio of 2:3

By section formula,

$$P(x,y) = P\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$\Rightarrow P(x,y) = P\left(\frac{2\times4+3\times4}{2+3}, \frac{2\times5+3\times(-5)}{2+3}\right)$$

$$\Rightarrow P(x,y) = P\left(\frac{20}{5}, \frac{-5}{5}\right) = P(4,-1)$$

$\therefore$ Coordinates of P are $\boxed{P(4,-1)}$

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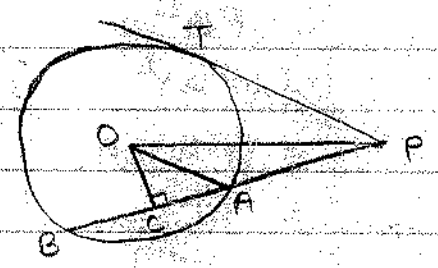
22. Given: Circle with centre O

PT is a tangent

$OC \perp AB$

To Prove: $PA \cdot PB = PC^2 - AC^2$



Proof: Perpendicular from centre to chord bisects it

$\Rightarrow$ C is midpoint of AB $\quad (\because \angle OCA = 90^\circ$, given)

$\Rightarrow BC = CA = \frac{AB}{2}$

$\Rightarrow AB = 2AC$

By Pythagoras Theorem,

In right $\triangle OCA$, $OA^2 = OC^2 + AC^2$

In right $\triangle OCP$, $OP^2 = OC^2 + PC^2$

$$
\begin{aligned}
LHS &= PA \cdot PB \\
&= PA \times (PA + AB) \\
&= PA^2 + PA \cdot AB \\
&= PA^2 + PA \cdot 2AC \qquad \text{(from ①)} \\
&= (PA)^2 + 2(PA)(AC) \\
&= (PA + AC)^2 - AC^2 \qquad \left[(a+b)^2 = a^2 + 2ab + b^2\right] \\
&= PC^2 - AC^2 = RHS
\end{aligned}
$$

Hence, proved

23. First number = 96
Second number = 120

$96 = 2^5 \times 3$

$120 = 2^3 \times 3 \times 5$

$HCF(96, 120) = 2^3 \times 3$

$= 8 \times 3$

$= 24$

2	96	2	120
2	48	2	60
2	24	2	30
2	12	3	15
2	6		5
	3		

$LCM(96, 120) = 2^5 \times 3 \times 5$

$= 32 \times 3 \times 5$

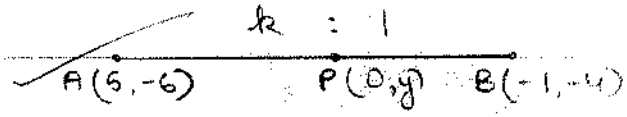
$= 480$

$\therefore$ HCF $(96, 120) = \boxed{24}$ and LCM $(96, 120) = \boxed{480}$

P.T.O.

24. Points are $A(5,-6)$ and $B(-1,-4)$

Let the point where y-axis $(x=0)$ intersects AB be $P(0,y)$



Let the ratio in which $P(0,y)$ divides AB be $k:1$

By section formula,

$$P(x,y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$\Rightarrow P(0,y) = \left(\frac{-k+5}{k+1}, \frac{-4k+(-6)}{k+1}\right)$$

$$\Rightarrow 0 = \frac{-k+5}{k+1} \quad \text{and} \quad y = \frac{-4k-6}{k+1}$$

$$\Rightarrow 0 = -k+5$$

$$\Rightarrow k = 5$$

$$\Rightarrow k:1 = 5:1$$

$\therefore$ Ratio is $\boxed{5:1}$

25. (a) $a\cos\theta + b\sin\theta = m$

$a\sin\theta - b\cos\theta = n$

To prove: $a^2 + b^2 = m^2 + n^2$

$a\cos\theta + b\sin\theta = m$

Squaring both sides,

$a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta = m^2$ $\left[(a+b)^2 = a^2 + b^2 + 2ab\right]$

$a\sin\theta - b\cos\theta = n$

Squaring both sides,

$a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta = n^2$ $\left[(a-b)^2 = a^2 + b^2 - 2ab\right]$

Adding ① and ②,

$(a^2\cos^2\theta + a^2\sin^2\theta) + (b^2\sin^2\theta + b^2\cos^2\theta) + 2ab\sin\theta\cos\theta - 2ab\sin\theta\cos\theta = m^2 + n^2$

$\Rightarrow a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = m^2 + n^2$

$\because \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow a^2 + b^2 = m^2 + n^2$

LHS = RHS

Hence, proved

SEC-C

26. (a) Let us assume, to the contrary, that $\sqrt{3}$ is rational.

$\Rightarrow \sqrt{3} = \frac{p}{q}$ where $q \neq 0$, p and q are coprime positive integers

Squaring both sides,

$\Rightarrow 3 = \frac{p^2}{q^2}$

$\Rightarrow p^2 = 3q^2$

$\Rightarrow$ 3 divides p^2

$\Rightarrow$ 3 divides p ($\because$ 3 is prime)

So, let $p = 3m$

Substituting in

$(3m)^2 = 3q^2$

$\Rightarrow 9m^2 = 3q^2$

$\Rightarrow q^2 = 3m^2$

$\Rightarrow$ 3 divides q^2

$\Rightarrow$ 3 divides q

From ③ and ④,

3 divides ~~both~~ p and q

But p and q are coprime, i.e. HCF$(p, q) = 1$ (Using ①)

which is a contradiction

$\therefore$ our supposition is wrong

$\therefore \sqrt{3}$ must be irrational.

Hence, proved

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27 Let the first term of AP be a

and common difference be d

AP: $a, a+d, a+2d, \ldots$

$a_n = a + (n-1)d$

$a_p = a + (p-1)d = q$

$\Rightarrow a + pd - d = q$

$a_q = a + (q-1)d = p$

$\Rightarrow a + qd - d = p$

Subtracting ① from ②,

$$a + qd - d = p$$
$$\ominus \quad a + pd - d = q$$
$$(q-p)d = p-q$$
$$\Rightarrow d = -1$$

Substituting in ①,

$$a + p(-1) - (-1) = q$$
$$\Rightarrow a + 1 = p + q$$

$$a_n = a + (n-1)d$$
$$= a + (n-1)(-1)$$
$$= a - n + 1$$
$$= (a+1) - n$$
$$= p + q - n$$

(Substituting from ③)

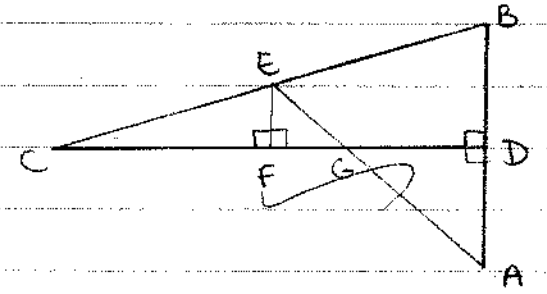
LHS = RHS

Hence, proved

28 (a) Given: CD is ⊥ bisector of AB

EF ⊥ CD

AE intersects CD at G



To Prove: $\frac{CF}{CD} = \frac{FG}{DG}$

Proof: In $\triangle EFG$ and and $\triangle ADG$,

$\angle EFG = \angle ADG = 90^\circ$ (given, linear pair with $\angle EFC$ and $\angle BDG$)

$\angle EGF = \angle AGD$ (vertically opposite angles)

$\therefore \triangle EFG \sim \triangle ADG$ (by AA similarity criterion)

$\Rightarrow \frac{EF}{AD} = \frac{FG}{DG}$ (cpst)

In $\triangle ECF$ and $\triangle BCD$,

$\angle EFC = \angle BDC = 90^\circ$ (given)

$\angle ECF = \angle BCD$ (common angle)

$\therefore \triangle ECF \sim \triangle BCD$ (by AA similarity criterion)

$\Rightarrow \frac{EF}{BD} = \frac{CF}{CD}$ (cpst)

But $AD = BD$ ($\because$ CD bisects AB)

$$\Rightarrow \frac{EF}{AD} = \frac{EF}{BD} = \frac{CF}{CD} \quad (2)$$

From (1) and (2),

$$\frac{EF}{AD} = \frac{CF}{CD} = \frac{FG}{DG}$$

$$\therefore \frac{CF}{CD} = \frac{FG}{DG}$$

Hence, proved

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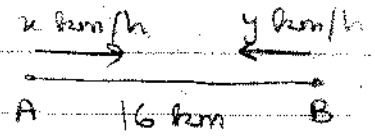
29. Let the speed of person 1 be x km/h
and speed of person 2 be y km/h $(x > y)$

distance = speed × time

Given distance = 16 km

Case 1 : Towards each other



Time = 2 h

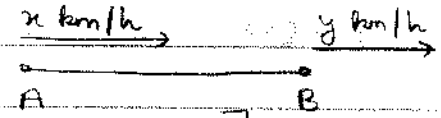
Speed = $(x+y)$ km/h

Distance = $2(x+y)$ km

$\Rightarrow \quad 16 = 2x + 2y$

$\Rightarrow \quad x + y = 8$

Case 2 : Same direction



Time = 8 h

Speed = $(x-y)$ km/h

Distance (only AB) = $8(x-y)$ $\quad [16 + 8y = 8x]$

$\Rightarrow \quad 16 = 8x - 8y$

$\Rightarrow \quad x - y = 2$

Adding ① and ②,

$$\begin{array}{r} x + y = 8 \\ \oplus \; x - y = 2 \\ \hline 2x = 10 \\ \hline \end{array}$$

$\Rightarrow x = 5$

Substituting in ①

$y = 3$ ~~km~~

$\therefore$ Speed of person 1 = $\boxed{5 \text{ km/h}}$

Speed of person 2 = $\boxed{3 \text{ km/h}}$

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30. $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$

$$\text{LHS} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} + \frac{1}{\tan\theta(1-\tan\theta)}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{\tan\theta(\tan\theta - 1)}$$

$$= \frac{\tan^3\theta - 1}{(\tan\theta - 1)\tan\theta}$$

$$= \frac{(\tan\theta - 1)(\tan^2\theta + 1 + \tan\theta)}{\tan\theta(\tan\theta - 1)}$$

$$\left[\begin{array}{l}(a - b)(a^2 + b^2 + ab) \\ = a^3 - b^3\end{array}\right]$$

$$= \frac{1 + \tan^2\theta + \tan\theta}{\tan\theta}$$

$$= \frac{\tan\theta + \sec^2\theta}{\tan\theta}$$

$$(1 + \tan^2\theta = \sec^2\theta)$$

$$= \frac{\tan\theta}{\tan\theta} + \frac{\sec^2\theta}{\tan\theta}$$

$$= 1 + \frac{1}{\cos^2\theta} \times \frac{\cos\theta}{\sin\theta}$$

$$\left(\begin{array}{l}\sec\theta = 1/\cos\theta \\ \tan\theta = \sin\theta/\cos\theta\end{array}\right)$$

$$= 1 + \frac{1}{\cos\theta\sin\theta}$$

$$= 1 + \sec\theta\,\mathrm{cosec}\,\theta$$

$$= \text{RHS}$$

Hence, proved

31

Classes	Frequency (f_i)	Midpoints (x_i)	$u_i = \frac{x_i - A}{h}$	$f_i u_i$
25 - 30	14	27.5	-3	-42
30 - 35	22	32.5	-2	-44
35 - 40	16	37.5	-1	-16
40 - 45	6	A 42.5	0	0
45 - 50	5	47.5	1	5
50 - 55	3	52.5	2	6
55 - 60	4	57.5	3	12
	$\Sigma f_i = 70$			$\Sigma f_i u_i = -79$

$$\text{Mean} = \bar{x} = A + \frac{h \Sigma f_i u_i}{\Sigma f_i}$$

$$= 42.5 + \frac{5 \times (-79)}{70}$$

$$= 42.5 - \frac{395}{70}$$

$$= 42.5 - 5.642$$

$$= 36.858 \approx 36.86$$

$\therefore$ Mean = 36.86 (approx)

SEC-D

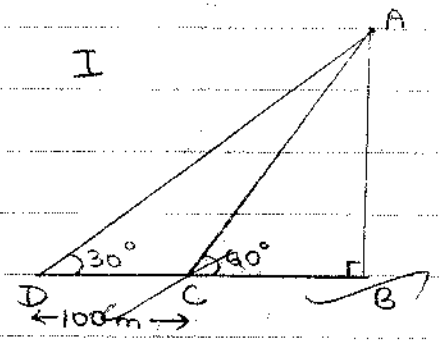
32. Introduction

2 cases are formed

A = hot-air balloon

C = first observer

D = second observer

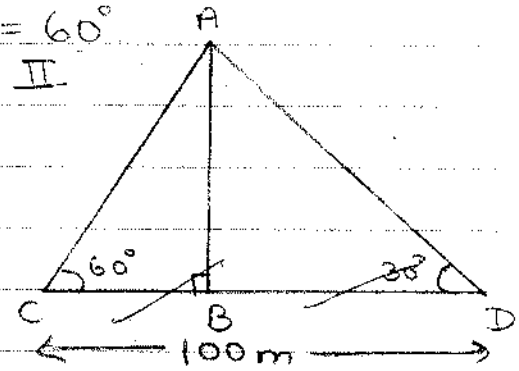


Angle of elevation of A from C = $\angle ACB = 60^\circ$

Angle of elevation of A from D

$= \angle ADB = 30^\circ$

CD = 100 m



(a). Let height of basket = AB = h m

Case I: In $\triangle ABC$,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{h}{BC}$$

$$\Rightarrow BC = \frac{h}{\sqrt{3}} \quad (1)$$

In $\triangle ABD$,

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{BC + 100}$$

$$\Rightarrow BC + 100 = h\sqrt{3}$$

$$BC + 100 = \frac{h\sqrt{3}}{}$$

$$\Rightarrow \frac{h}{\sqrt{3}} + 100 = h\sqrt{3}$$

$$\Rightarrow 100 = h\sqrt{3} - \frac{h\sqrt{3}}{3}$$

$$\Rightarrow \frac{2h\sqrt{3}}{3} = 100$$

$$\Rightarrow h = \frac{150}{\sqrt{3}} = 50\sqrt{3}\ m$$

$\therefore$ Height of basket $= \boxed{50\sqrt{3}\ m}$

<u>Case 2</u>: In ΔABC,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{h}{BC}$$

$$\Rightarrow BC = \frac{h}{\sqrt{3}}$$

In ΔABD,

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{BD}$$

$$\Rightarrow BD = h\sqrt{3}$$

$BC + BD = 100\,m$

$\Rightarrow \frac{h}{\sqrt{3}} + h\sqrt{3} = 100$

$\Rightarrow \frac{4h\sqrt{3}}{3} = 100$

$\Rightarrow h = 25\sqrt{3}$

$\therefore$ Height of basket $= \boxed{25\sqrt{3}\,m}$

(b). Case 1 Dist of A from C = AC

In $\triangle ABC$,

$\sin 60^\circ = \frac{AB}{AC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{50\sqrt{3}}{AC}$

$\Rightarrow AC = 100\,m$

$\therefore$ Distance of basket from first observer $= \boxed{100\,m}$

Case 2: Dist. of A from C = AC

In $\triangle ABC$,

$$\sin 60^\circ = \frac{AB}{AC}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{25\sqrt{3}}{AC}$$

$$\Rightarrow AC = 50\,m$$

$\therefore$ Distance of basket from first observer = $\boxed{50\,m}$

(c) Case 1 To find - BD

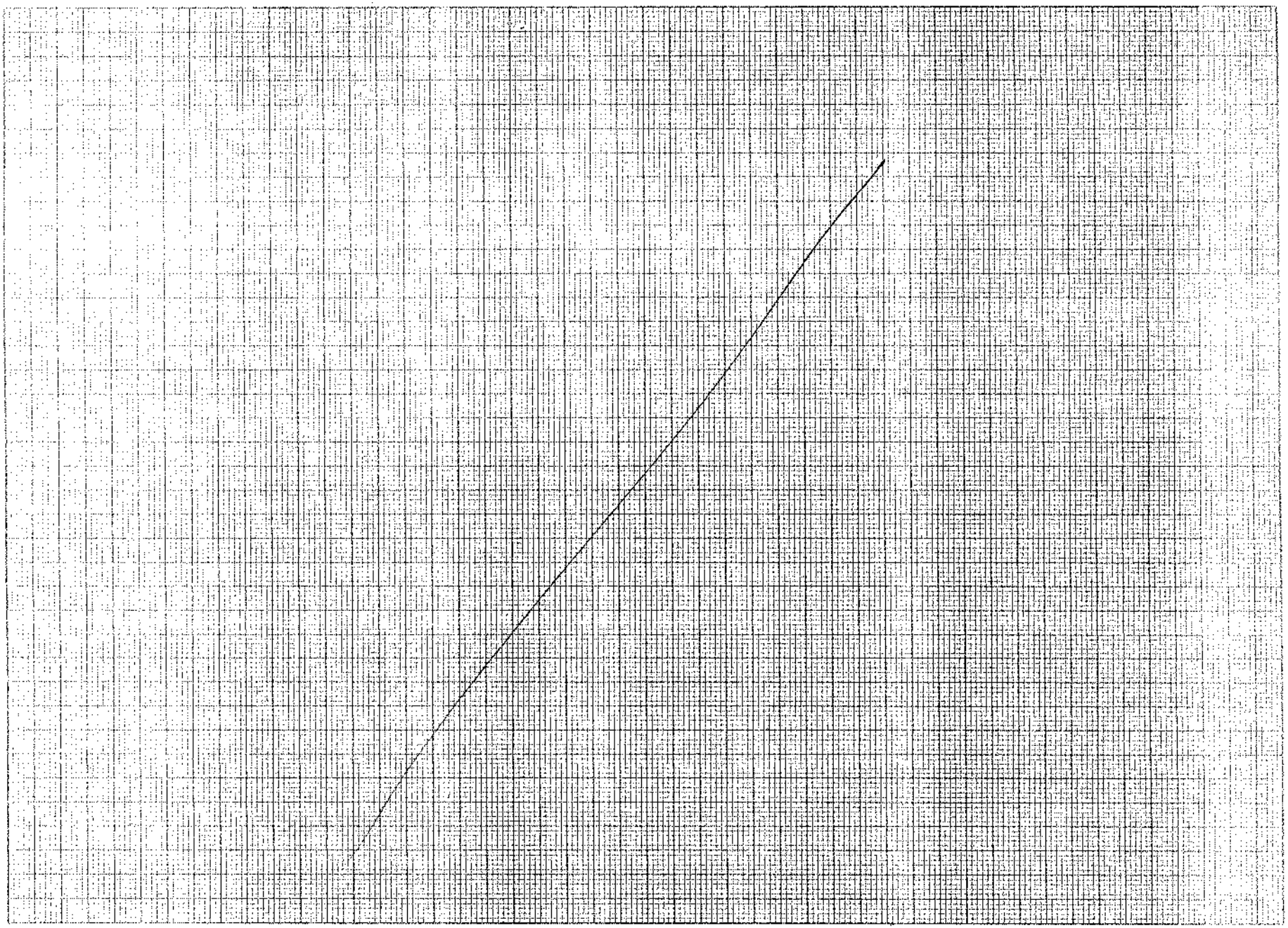
$$BD = BC + CD$$

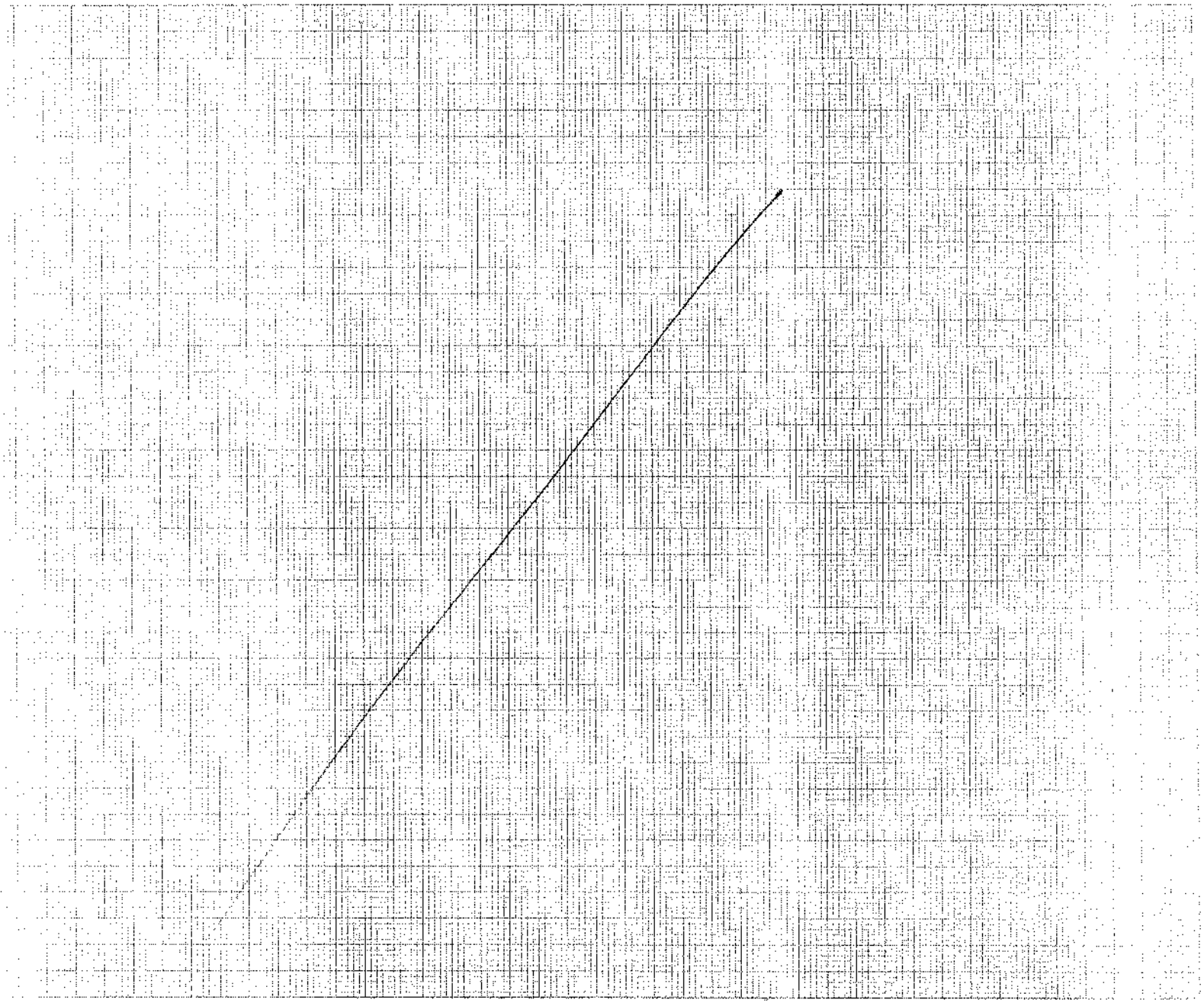
$$= \frac{h}{\sqrt{3}} + 100 \qquad \text{(from ①)}$$

$$= \frac{50\sqrt{3}}{\sqrt{3}} + 100$$

$$= 50 + 100 = 150$$

$\therefore$ Horizontal distance BD = $\boxed{150\,m}$





Case 2. To find – BD

$BD = h\sqrt{3}$ (from ②)

$= 25\sqrt{3} \times \sqrt{3}$

$= 75$

∴ Horizontal distance BD = $\boxed{75\ m}$

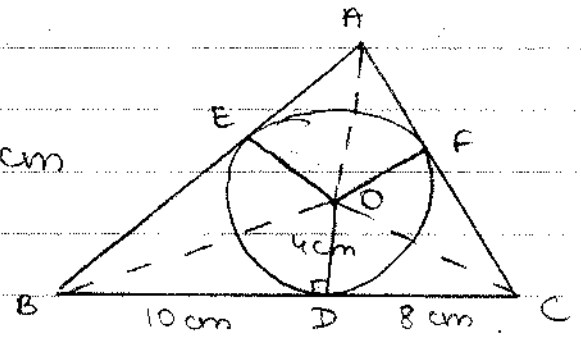
3.3 (a.) Given: $\triangle ABC$

Incircle with radius $r = 4$ cm

BD = 10 cm

CD = 8 cm

ar(ABC) = 90 cm^2



To find: Lengths of AB and AC.

Construction: Join OE, OF, OA, OB, OC

OD = OE = OF = r = 4cm (radii)

Tangents from same external point are equal in length.

From pt. A, $AE = AF = x$ (let it be)

From pt. B, $BE = BD = 10$ cm

From pt. C, $CD = CF = 8$ cm

Area of $\Delta = \frac{1}{2} \times b \times h$

Area of ΔABC = Area of ΔOAB + Area of ΔOBC + Area of ΔOCA

$$\Rightarrow \quad 90 = \frac{1}{2} \times OE \times AB + \frac{1}{2} \times OD \times BC + \frac{1}{2} \times OF \times AC$$

$$\Rightarrow \quad 90 = \frac{1}{2} \times r \times (AE + BE) + \frac{1}{2} \times r \times (BD + CD) + \frac{1}{2} \times r \times (CF + AF)$$

$$\Rightarrow \quad 90 = \frac{1}{2} \times r \times (x + 10) + \frac{1}{2} \times r \times (10 + 8) + \frac{1}{2} \times r \times (8 + x)$$

$\Rightarrow 90 = \frac{1}{2} \times 4 \left(x + 10 + 10 + 8 + 8 + x\right)$

$\Rightarrow 45 = 2x + 36$

$\Rightarrow 2x = 9$

$\Rightarrow x = 4.5$

$AB = AE + BE = 4.5 + 10 = 14.5\text{ cm}$

$AC = AF + CF = 4.5 + 8 = 12.5\text{ cm}$

$\therefore AB = \boxed{14.5\text{ cm}}$

$AC = \boxed{12.5\text{ cm}}$

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34. Let the original average speed be x km/h.

<u>Original</u>: distance = 54 km

speed = x km/h

time = $\frac{\text{distance}}{\text{speed}} = \frac{54}{x}$ h

New. distance = 63 km

speed = $(x+6)$ km/h

time = $\frac{\text{distance}}{\text{speed}} = \frac{63}{x+6}$ h

$$\frac{54}{x} + \frac{63}{x+6} = 3$$

$$\Rightarrow 3\left(\frac{18}{x} + \frac{21}{x+6}\right) = 3$$

$$\Rightarrow \frac{18x + 108 + 21x}{x^2 + 6x} = 1$$

$$\Rightarrow 39x + 108 = x^2 + 6x$$

$$\Rightarrow x^2 - 33x - 108 = 0$$

$$\Rightarrow x^2 - 36x + 3x - 108 = 0$$

$$\Rightarrow x(x-36) + 3(x-36) = 0$$

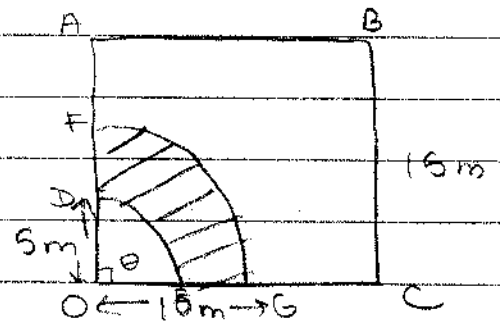
$$\Rightarrow (x-36)(x+3) = 0$$

$$\Rightarrow x = 36 \text{ or } x = 3$$

$\because$ speed cannot be negative,
$x = -3$ will be neglected.
$\Rightarrow x = 36$

$\therefore$ Average speed of train (original) = $\boxed{36 \text{ km/h}}$

35. Square field OABC
Side = s = 15 m



Quadrant
$\angle DOE = \theta = 90^\circ$ (all angles of square $= 90^\circ$)
Radius = length of rope
$= r = 5$ m

Area grazed by horse = Area of sector DOE
$= \dfrac{\theta}{360^\circ} \times \pi \times r^2$

$$= \frac{90}{360} \times \frac{3.14}{100} \times 5 \times 5$$

$$= \frac{3925}{200}$$

$$= \boxed{19.625\ m^2}$$

$$\begin{array}{r} 157 \\ 25 \\ \hline 785 \\ 314\times \\ \hline 3925 \\ 2 \end{array}$$

$= 1962.5$

Now, new radius = new length of rope = R = 10m

Increase in grazing area = Area of sector FOG − Area of sector DOE

$$= \frac{\theta}{360^\circ} \times \pi \times R^2 - \frac{\theta}{360^\circ} \times \pi \times r^2$$

$$= \frac{90}{360} \times \frac{314}{100} \times (10^2 - 5^2)$$

$$= \frac{90}{360} \times \frac{314}{100} \times 15 \times 5$$

$$\boxed{a^2 - b^2 = (a+b)(a-b)}$$

$$= \frac{11775}{200}$$

$= \boxed{5.8875\ m^2}$

Original area grazed $= \boxed{1.9625\ m^2}$

Increase in area $= \boxed{5.8875\ m^2}$

SEC-E

36 Golf ball = Sphere

Radius $= R = \frac{4.2}{2} = 2.1$ cm

Dimple = Hemisphere

Radius $= r = 2$ mm $= 0.2$ cm

(i) SA of 1 dimple = CSA of hemisphere

$= 2\pi r^2$

$= 2 \times \frac{22}{7} \times \frac{2}{10} \times \frac{2}{10}$

$$= \frac{176}{700} \text{ cm}^2$$

$$= \boxed{0.2514 \text{ cm}^2}$$

(ii) Vol to make 1 dimple = Vol. of hemisphere

$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{2}{10} \times \frac{2}{10} \times \frac{2}{10}$$

$$= \frac{352}{21000}$$

$$= \boxed{0.01676 \text{ cm}^3}$$

21000) 352.0 (0.0167
35200
21000
142000
126000
160000
147000
130000

(iii) (b) Vol of golf ball = Vol of sphere − Vol of 315 hemispheres

$$= \frac{4}{3}\pi R^3 - \frac{2}{3}\pi r^3 \times 315$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{10} \times \frac{21}{10} \times \frac{21}{10} - \frac{2}{3} \times \frac{22}{7} \times \frac{2}{10} \times \frac{2}{10} \times \frac{2}{10} \times 315$$

$= 38.808 - 5.28$

$= \cancel{33.688}\ \cancel{33.588}\ \boxed{33.528\ cm^3}$

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37 (i) Spinner I - Spinner II

Red (R) - Red (R) { RR,
Red (R) - Blue (B) RB,
Red (R) - Green (G) RG,
Green (G) - Red (R) GR,
Green (G) - Blue (B) GB,
Green (G) - Green (G) GG,
Yellow (Y) - Red (R) YR,
Yellow (Y) - Blue (B) YB,
Yellow (Y) - Green (G) YG }

Total no. of outcomes = $\boxed{9}$

(ii) $X = \{RB\}$

Favourable outcomes = 1

$P(\text{making purple}) = \dfrac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}} = \boxed{\dfrac{1}{9}}$

(iii) (a) No. of participants = 99

Winning

No. $= \frac{1}{9} \times 99 = 11$

Amount = ₹(−10) (school has to pay)

Value = ₹(−110) [loss]

Loss

No. = 99 − 11 = 88

Amount = ₹5 (pay to school)

Value = ₹440 [gain]

Net = + 440 − 110

= + 330

∴ So, the school most likely collected $\boxed{\text{Rs. } 330}$

35. $p(t) = 20t - 16t^2$

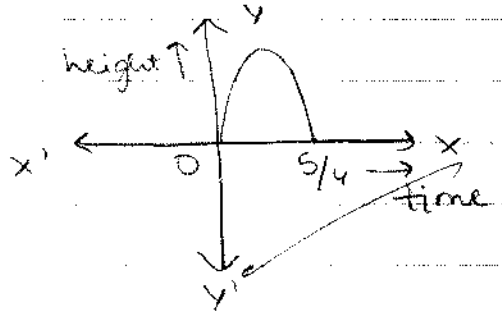
(i) $20t - 16t^2 = 0$

$\Rightarrow -4t(4t - 5) = 0$

$\Rightarrow t = 0$ or $t = \frac{5}{4}$

$\therefore$ Zeroes of $p(t) = 20t - 16t^2$ are $\boxed{0 \text{ and } \frac{5}{4}}$

(ii) (a)



→ opens downwards $(a < 0)$

→ intersects x-axis at $(0, 0)$ and $\left(\frac{5}{4}, 0\right)$

(iii) Water level is hit when $h = 0$

(b) $h = 20t - 16t^2$

$\Rightarrow 0 = -4t(4t - 5)$

$\Rightarrow t = 0$ or $t = \frac{5}{4}$

$\because$ Delphin has started at $t = 0$

$\Rightarrow$ at $t = \frac{5}{4}$, delphin reaches water level again.

distance covered = speed × time

$= 20\ cm/s \times \frac{5}{4}\ s$ 2

$= \boxed{25\ cm}$

END :)